Name _____ Class _____ Date _____

Reteaching

Polynomials, Linear Factors, and Zeros

The Factor Theorem tells you that if you know the zeros of a polynomial function, you can write the polynomial.

Factor Theorem

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Problem

What is a cubic polynomial function in standard form with zeros 0, 4, and -2?

Each zero (a) is part of a linear factor of the polynomial, so you can write each factor as $(x - a)$.

$(x - a_1)(x - a_2)(x - a_3)$	Set up the cubic polynomial factors.
$a_1 = 0,\ a_2 = 4,\ a_3 = -2$	Assign the zeros.
$(x - 0)(x - 4)[x - (-2)]$	Substitute the zeros into the factors.
$f(x) = x(x - 4)(x + 2)$	Write the polynomial function in factored form.
$f(x) = x(x^2 - 2x - 8)$	Multiply $(x - 4)(x + 2)$.
$f(x) = x^3 - 2x^2 - 8x$	Multiply by x using the Distributive Property.

The polynomial function written in standard form is $f(x) = x^3 - 2x^2 - 8x$.

Exercises

Write a polynomial function in standard form with the given zeros.

1. $5, -1, 3$

2. $1, 7, -5$

3. $-1, 1, -6$

4. $2, -2, -3$

5. $2, 1, 3$

6. $2, 3, -3, -1$

7. $0, -8, 2$

8. $-10, 0, 2$

9. $-2, 2, -\frac{3}{2}$

10. $-1, \frac{2}{3}$

Name _____ Class _____ Date _____

Reteaching (continued)

Polynomials, Linear Factors, and Zeros

You can use a polynomial function to find the minimum or maximum value of a function that satisfies a given set of conditions.

Problem

Your school wants to put in a swimming pool. The school wants to maximize the volume while keeping the sum of the dimensions at 40 ft. If the length must be 2 times the width, what should each dimension be?

Step 1 First, define a variable x. Let $x =$ the width of the pool.

Step 2 Determine the length and depth of the pool using the information in the problem.

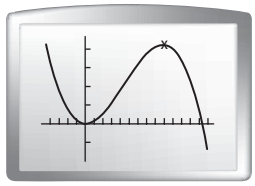
The length must be 2 times the width, so length $= 2x$.

The length plus width plus depth must equal 40 ft, so depth $= 40 - x - 2x = 40 - 3x$.

Step 3 Create a polynomial in standard form using the volume formula

$$\begin{aligned} V &= \text{length} \cdot \text{width} \cdot \text{depth} \\ &= 2x(x)(40 - 3x) \\ &= -6x^3 + 80x^2 \end{aligned}$$

Step 4 Graph the polynomial function. Use the MAXIMUM feature. The maximum volume is 2107 ft^3 at a width of 8.9 ft.



Maximum
X = 8.8888882 Y = 2106.9959

Step 5 Evaluate the remaining dimensions: width $= x \approx 8.9$ ft

length $= 2x \approx 17.8$ ft

depth $= 40 - 3x \approx 13.3$ ft

Exercises

11. Find the dimensions of the swimming pool if the sum must be 50 ft and the length must be 3 times the depth.

12. Find the dimensions of the swimming pool if the sum must be 40 ft and the depth must be one tenth of the length.

13. Find the dimensions of the swimming pool if the sum must be 60 ft and the length and width are equal.